Stereoscopic Matchmoving in 3DE4

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Version 2: 2009-08-12, Rotation policy
Version 3: 2009-08-14, Zoom
Version 4: 2009-08-18, Interocular
Version 5: 2009-09-15, English and completely restructured
Version 6: 2009-10-02, Object Point Group, Depth Shift
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Contents

1 *Introduction*

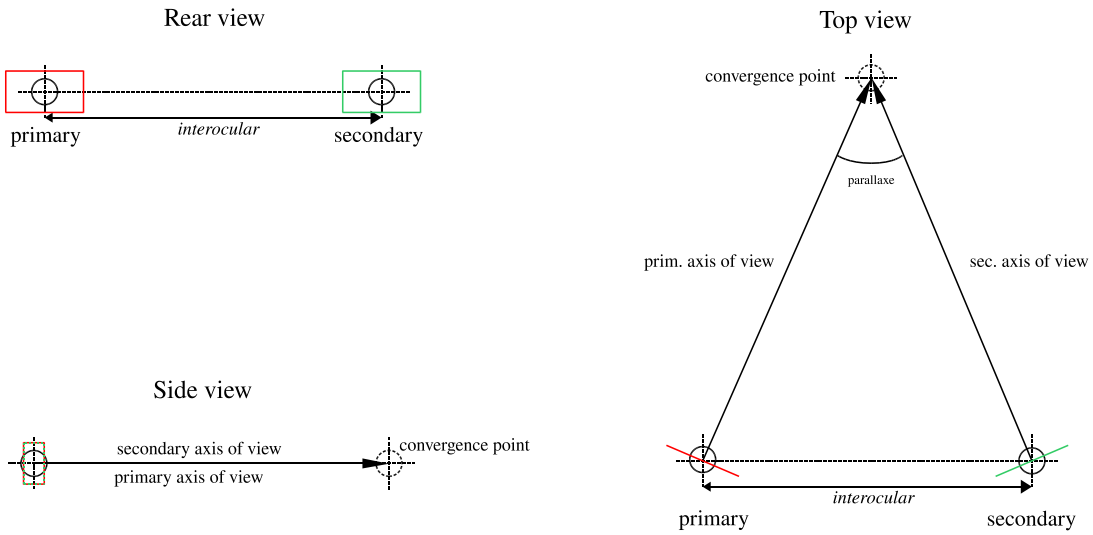
The purpose of this document is to fix some definitions and terminology we use in 3DE4 and to give an overview how stereoscopic projects can be done. This document is not meant to be a tutorial, but to provide some additional information which usually is not part of a tutorial. It is addressed to experienced users. The document refers to 3DE4 release 1 beta 6.

2 *Definitions*

In the following we will juxtapose the ideal stereo rig and it's realistic counterpart. A realistic stereo has (usually unintentionally) more degrees of freedom than the ideal one, and 3DE4 should be able to account for this. Some of the additional degrees of freedom are due to physical impact, others are a inherent to construction of the rig.

2.1 The ideal Stereo Rig

We consider a stereo rig to be ideal when it is constructed in an exactely symmetric way. The optical axes of both rig intersect precisely in a point which is called *convergence point*. The projection centers of both camera have the same distance to the *convergence point*. The right vectors of both cameras lie in within the stereoscopic plane. As a consequence the up-vectors are parallel. *Interocular* and *parallaxe* vary in a well-defined way during the shot and are smooth functions over time. When zoom lenses are used, the zoom factors of both cameras are equal at each time.



2.1.1 *Lens distortion*

Lens distortion is basically not a draw back of the ideal stereo rig, as long as it is always the same for both cameras (which in practice is not the case). When lens distortion depends on other parameters like zoom (or even parallaxe), the dependency must be the same for both cameras. When this is granted, lens distortion can easily be compensated, and we still consider the rig as ideal.

2.2 A realistic Stereo Rig

According to our experience which is based on reports from users, in a real scenario there can be undesired effects, which we will describe now. For any given time during the shot the following can occur:

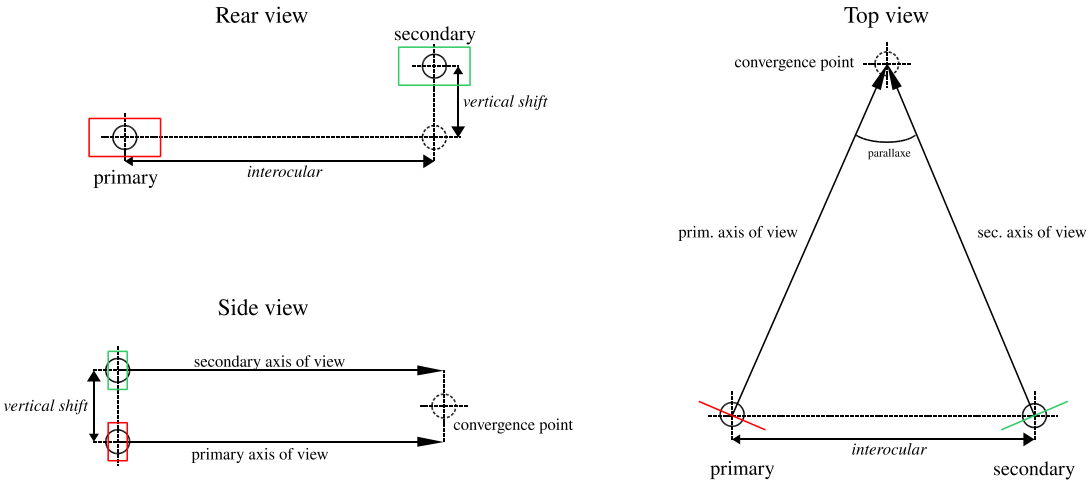
- The two cameras are not at the same height level (→ *vertical shift*)
- The optical axes do not intersect (i.e. they are *skew*. This affects the definition of *convergence*)
- The right vector of at least one of the cameras is not parallel to the stereoscopic plane (→ *rotation policy*)
- The two cameras have different optical properties like focal length, lens distortion (→distinct lens objects in 3DE4)
- The two cameras are not at the same depth level (→ *depth shift*). This is an artefact of asymmetric construction e.g. by means of mirrors.

Additionally, the following time-dependent effects can occur:

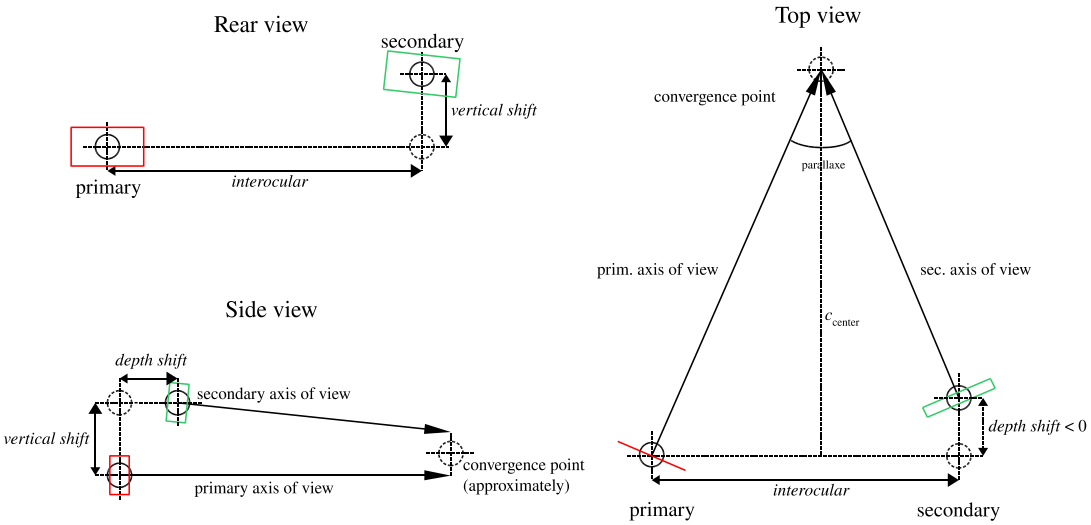
- In case of zooming, the cameras have different zoom factors (→ *zoom policy*). Since both camera have their own zoom lenses, zoom factors will be equal only up to a certain precision. However, a different zoom factor has direct impact on the recorded footage. According to our considerations a difference of 0.5% in focal length will result in up to 5 pixel difference in the image (at 2k image size).
- lens distortion can depend on zoom or other parameters in a different way for both

cameras. The dependency between lens distortion and zoom is solved in 3DE4 by using distinct lens objects for both cameras, but doing so requires that reference measurements (grid shots) for zoom and lens distortion are available for both cameras.

The following figure illustrates our definition of vertical shift and the convergence point in presence of vertical shift. The rotation policy in this case is *y*, which means, both axes of view and right vectors rotate within parallel planes.



In the next figure you see a combination of vertical shift, depth shift and rotation policy *xyz*. Note, that the axis of view and the right vector of the secondary camera do not rotate within the stereoscopic plane.



3 *Stereoscopy in 3DE4*

The task of 3DE4 is to find a consistent matchmove solution for both cameras of the rig and all point groups, based on parameters and policies given by the user.

3.1 Basic Setup

A stereoscopic project in 3DE4 consists of two stereo cameras (left one and right one) and possibly other non-stereoscopic camera objects like reference cameras or further sequence cameras. Most of the camera properties (exception: zoom policy) are specified in

Attribute Editor → Cameras → Stereoscopic

Stereoscopic calculations are basically done by 3DE4's core in two distinct passes: In the first pass the path of one of the two stereo cameras is calculated with respect to the camera point group (along with reference cameras and non-stereoscopic cameras) if there is one. We call this the *primary camera*. Then the other stereoscopic camera is added and both camera paths are optimized simultaneously. We call this the *secondary camera*. While the *primary camera* must be fully equipped with tracking data, this is not required for the *secondary camera*, although it is highly recommended to provide comprehensive tracking data for the latter as well. As part of the basic setup the user specifies which of the stereoscopic camera is *primary* and which is *secondary*, and which of the cameras is *left-hand* and which is *right-hand*. So, every stereoscopic project contains a set of cameras with the following properties:

primary - left-hand
secondary - right-hand

or

primary - right-hand
secondary - left-hand.

When both cameras are equipped with a sufficient amount of tracking data, you can disable either of them separately, turning the project into a standard non-stereoscopic project. This is important for trouble-shooting ("divide-and-conquer").

3.1.1 *Classification of 3DE4-projects*

We consider a project as *more complex*, if there are more occasions to make mistakes. This applies to the user as well as the developer. In so far we come to the following hierarchy of complexity:

Non-stereoscopic projects
↓
Stereoscopic projects with one point group
↓
Stereoscopic projects with more than one point group

We mention this because it reflects the way to get along when problems occur. Clearly, our classification is simplified. More classification could be added, and the result would not be a simple sequence like this one. But the general rule is:

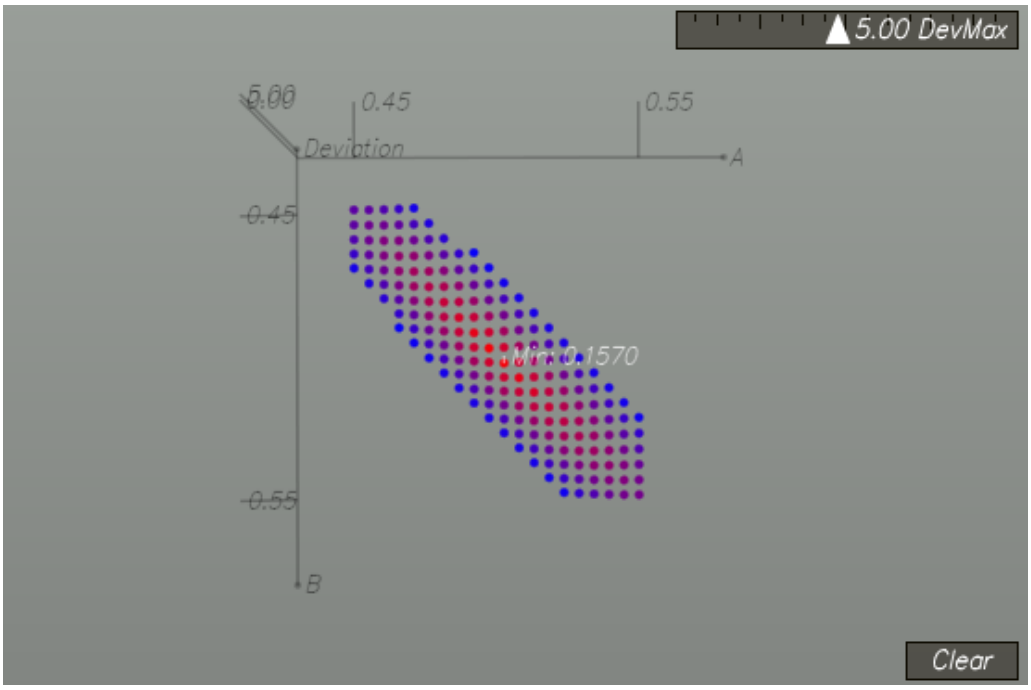
If there is trouble with some complex project, reduce it to a simpler one until you find the error.

This means, for instance, to disable a point group or to disable either primary or secondary camera. The same hierarchy can be used for building a project; you start with a simple, non-stereoscopic setup. When this works fine, add a camera and specify primary and secondary.

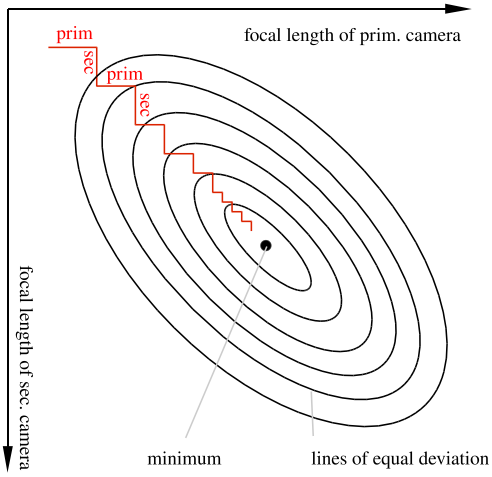
If this works as well add the second point group. Of course, in real-life production often there is little time for experiments, but for less experienced users, we consider this the best way.

3.1.2 *Single lens vs. separate lenses*

In setting up a stereo project at some point you will decide if both camera objects are connected to the *same* or to *two different* lens objects. Clearly, for modelling an ideal rig one lens object is used for both cameras. In practice however, there are good reasons to use a distinct lens for each camera. Generally, we do not know in how far the physical properties of the left and right camera coincide. Experience shows that suspiciousness is advisable. The drawback of using a second lens is, that you have at least one parameter more to adjust, namely the focal length. In contrast to non-stereoscopic projects there is an addidional aspect which we will discuss in the following. When you adjust both focal lengths using the brute force method, you might get a result like the following (top view):



The red dots mark the area around the minimum (of 0.157 pixel in this example). Here you see that both focal lengths are correlated. Let us assume you adjust both focal lengths separately by fine adjustment - primary camera, then secondary, then primary again, etc. Then the following will happen:



Slow convergence when focal lengths are optimized separately.

The red line represents the path of the system during parameter adjustment. Horizontal segments mean the primary focal length is adjusted, vertical lines represent adjustments of the secondary camera. Each of the line segments stands for a complete one parameter adjustment with, say, 20 attempts. The entire procedure will converge very slowly. This leads us to some cooking recipe for adjusting focal lengths:

In stereoscopic projects with two lens objects adjust both focal lengths simultaneously, not separately.

This rule applies for both, the brute force method as well as the adaptive method. Adjusting both focal lengths simultaneously is a gain in performance and precision. There is however another way to obtain both focal lengths. You can e.g. disable the secondary sequence and do a parameter adjustment with the primary camera only. Then turn on the secondary camera again and do a **fine** or **custom** parameter adjustment with the secondary focal length. This will also lead to a reasonable result, but if you really want to be sure to find the optimum values, the method of choice is to adjust both focal lengths simultaneously as described before.

3.2 Parameters and Policies

This section describes where the various parameters and policies are specified within 3DE4's user interface.

3.2.1 *Interocular*

As described before, by *interocular* we understand the distance of the camera projection centers in an ideal rig. In the realistic model it is the length of the distance vector of the two camera positions projected into the stereoscopic plane and projected into the plane perpendicular to the central direction of view. 3DE4 allows various modes for calculating or specifying this parameter. *Interocular* can be either time-independent ("static") or time-dependent ("dynamic"). We will discuss the modes in detail. The interocular mode is specified in

Attribute Editor → Cameras → Stereoscopic → Interocular

It is associated to the primary camera, i.e. you can edit it when the primary camera is selected. There are four modes:

Static - User Defined

Static - Calculated

Static - Adjust

Dynamic - User Defined

These modes have the following effects:

- *Static - User Defined* - Interocular is not time-dependent. "User defined" means the value is directly entered in the text field right to the option menu. When interocular is prescribed by production or predefined for instance by a data sheet this mode makes sense. Whenever stereoscopic calculations are performed, this interocular is used in aligning the secondary camera.
- *Static - Calculated* - In this mode, interocular is not time-dependent as well, but calculated internally whenever the stereoscopic calculations are performed. This mode makes sense, when no survey data are available, i.e. the scaling of the point group is completely undefined. 3DE4 will fix the scaling at some point during the calculation, but since scaling is directly correlated with interocular, 3DE4 needs to calculat interocular as well, which is the purpose of this mode.
- *Static - Adjust* - Interocular is not time-dependent. When this is selected, a panel for adjusting *interocular* appears in the *Parameter Adjustment Window*. In this case interocular is optimized along with other parameters e.g. focal length or whatever parameter the user wants to be optimized.
- *Dynamic - User Defined* Interocular is time-dependent. The current release of 3DE4 isn't yet able to calculate dynamic interocular. Therefore, the interocular curve needs to be edited by hand or imported by script. In practice, sometimes interocular curve can be retrieved from metadata of the camera rig, but lacks precision or is superimposed by other effects, which often makes it necessary to apply some filter or tweak it by hand. In order to import an interocular curve for setting *Dynamic - User Defined*, the following script (or a similar one) can be used. It expects a data file with two columns, one for the frame index and one for the interocular value for that frame.

```
import string

# This script imports an interocular curve
# into the currently selected camera object.

id_camera = tde4.getCurrentCamera()
id_curve = tde4.getCameraStereoInterocularCurve(id_camera)

# delete current curve
tde4.deleteAllCurveKeys(id_curve)

filename = tde4.postFileRequester("Select data file","*")

# open will throw if this fails
f = open(filename,"r");

for line in f:
        cols = string.split(line," ")
        tde4.createCurveKey(id_curve,[float(cols[0]),float(cols[1])])
```

```
        f.close()
```

Script for importing dynamic interocular

3.2.1.1 *Static adjust* vs. *Static calculated*

In principal, the modes *Static - Adjust* and *Static - Calculated* should lead to the same value for interocular. In practice, this is not always the case, since the methods to calculate interocular are different for these modes. Strictly speaking *Static - Adjust* optimizes directly the deviation (→ *Deviation Browser*) while *Static - Calculated* does not. Since the deviation is only weakly dependent of interocular in many projects the discrepancy can be a few percent. On the other hand, the brute force method in *Static - Adjust* scans the parameter space with some finite step size, while *Static - Calculated* uses an adaptive method with much higher precision. If you are doing a parameter adjustment with two or more other parameters, it makes sense to use *Static - Calculated* instead of *Static adjust* because this reduces the number of parameters in the Parameter Adjustment Window and thus increases performance.

3.2.1.2 Sign convention

As mentioned in the previous section, you have to specify, which of the two cameras is *left-hand* and which is *right-hand*. By this you define the sign convention of interocular. A *positive* value for interocular means, the cameras are positioned as you specified in the setup of the project (i.e. the right-hand camera appears on the right hand side, and the left-hand camera on the left hand-side), while a *negative* value swaps the camera positions. If you let 3DE4 calculate interocular, the system may find out, that the arrangement of the two cameras was wrong (i.e. left and right exchanged by mistake). In this case interocular will be negative, but the project can be calculated nonetheless.

3.2.2 *Vertical Shift*

The current stereo rig model of 3DE4 allows compensation for static (i.e. not time-dependent) *vertical shift*. The vertical shift mode is specified in

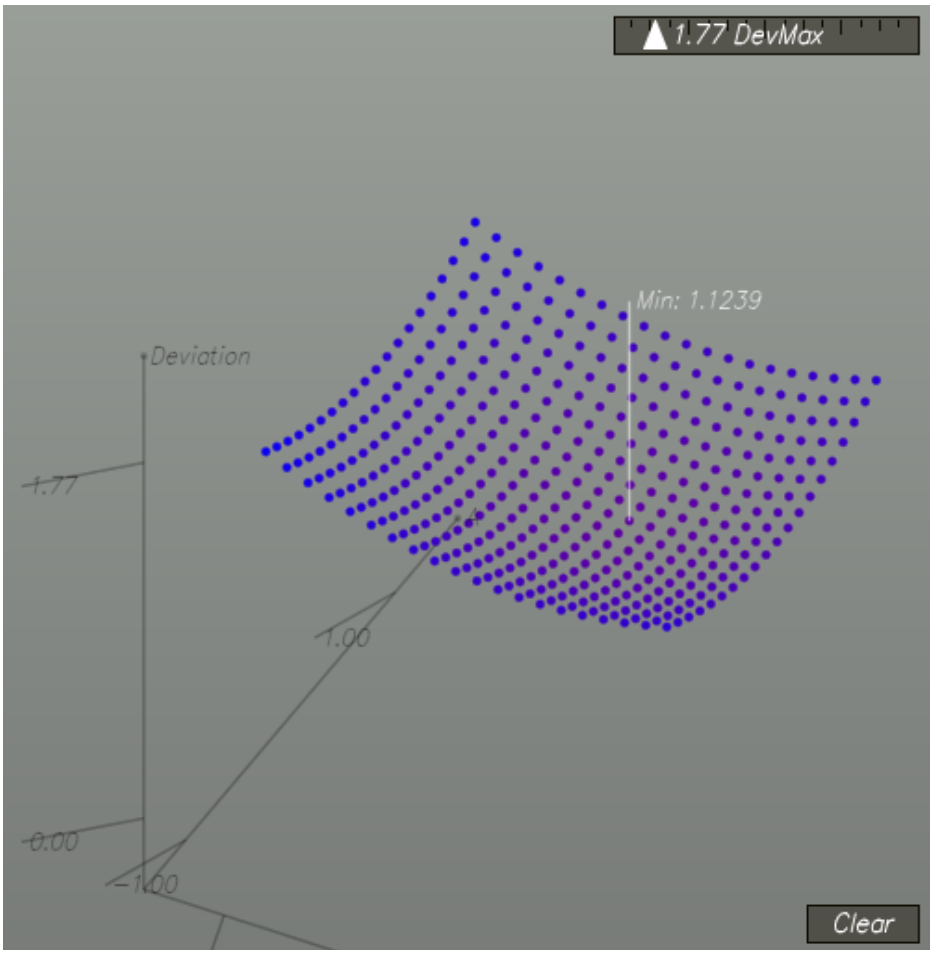
Attribute Editor → Cameras → Stereoscopic → Vertical Shift

In the user interface it is associated to the primary camera, i.e. you can edit it when the primary camera is selected. The two possible settings are:

Static - User Defined

Static - Adjust

When the menu is set to *Static - User Defined*, 3DE4 uses the value in the accompanying text field. When set to *Static - Adjust*, a panel for this parameter appears in the *Parameter Adjustment Window* and will be calculated by 3DE4. A positive value means, that the secondary camera is shifted in the positive *y*-axis expressed in coordinates of the primary camera. In parameter adjustment it turns out that the deviation often shows only a weak but clear dependency on vertical shift. Therefore, the value can either be optimized easily, or it is likely to be irrelevant and should be left zero. The following figure shows the result of a two parameter adjustment of interocular and vertical shift in the range between 2.8cm and 4.8cm for interocular and -1.0cm and +1.0cm for vertical shift.



Deviation vs. interocular and vertical shift

3.2.3 *Depth Shift*

The current stereo rig model of 3DE4 allows compensation for static (i.e. not time-dependent) *depth shift*. The depth shift mode is specified in

Attribute Editor → Cameras → Stereoscopic → Vertical Shift

In the user interface it is associated to the primary camera, i.e. you can edit it when the primary camera is selected. The two possible settings are:

Static - User Defined

Static - Adjust

3.2.4 *Rotation Policy*

By *rotation policy* we understand the behaviour of the rotational degrees of freedom of the secondary camera. In the ideal rig both cameras can be rotated towards each other around their *y*-axis and 3DE4 reproduces this behaviour in stereo calculations. However, experience shows, that the secondary camera also rotates around the other axes as well. We assume that this is due to mechanical slackness or imprecise construction of the rig. In order to handle this effect, there is a rotation policy which allows the secondary camera to rotate around all three axes. In principle more rotation policies are imaginable, for instance rotation around a constant axis that does not conincide with the *y*-axis. The current release of 3DE4 is restricted to the most important policies, namely time-dependent rotations around *y* and *xyz*. The *rotation policy* is specified in

Attribute Editor → Cameras → Stereoscopic → Rotation Policy.

By definition (and mainly for compatibility reasons) we associate this property to the secondary camera. There are two settings:

Allow y-Rotation Only

Allow Rotation Around all Axes

which correspond to our *rotation policies y* and *xyz*, respectively. In *rotation policy y* the optical axes of both cameras are parallel to the stereoscopic plane. In *rotation policy xyz* they can be skew. Mostly, when *vertical shift* is non-zero, you will use *rotation policy xyz*.

3.2.5 *Zoom Policy*

In contrast to rotation policy, zoom policy is not specified as a parameter in some option menu. Rather, it is specified by the user exactly at the point when it comes to calculating the zoom curve (or zoom curves). In an ideal rig both camera have the same focal length. In zoomed shots they also have the same time-dependent focal length, i.e. their zoom changes synchronuously. However, it has turned out in practice, that this is not always the case. This can be due to slackness or thermal deformation, since clearly we are talking about a mechanical system that is exposed to various kinds of physical influences on set, like solar (thermic) radiation, humidity, mechanical impact, abrasion etc. (we can only guess). Therefore we need to be able to calculate zoom for both primary and secondary camera separately. Moreover, it might be attractive to have a tool which tells you exactly, in how far the zoom factors differ, since this information might be helpful in order to correct either primary or secondary sequence. By appropriate scripts, 3DE4 will give out this time-dependent ratio of primary and secondary zoom. As in non-stereoscopic projects calculating the zoom curve is done in

Curve Editor → Calc → Zoom Curve → Calc Zoom Curve From Scratch

Curve Editor → Calc → Zoom Curve → Finetune Zoom Curve From Scratch

When you select one of these functions in a stereoscopic project 3DE4 will ask you to decide whether both stereo cameras should be calculated with *same* or *different* zoom factor. The choice depends on the stereo rig's construction and the above mentioned factors that might affect mechanical precision of the rig. Although the zoom of the primary camera can exhibit statistic and systematic errors (depending on the project setup), we think that the *relative* zoom factor between primary and secondary camera is reconstruct very precisely. In other words, the errors of primary and secondary zoom curve are highly correlated, while their ratio is calculated precisely. We have discussed this effect for static focal length in section 3.1.2. Since the ratio of the primary and secondary zoom curve is precise, we think that this could be used in order to apply zoom correction in the footage, although we do not have reports, that this is in fact done in practice. Once zoom is calculated, you can export the corresponding curve with some python script, like the following:

```
# This script exports the zoom curve
# of the currently selected camera object.

id_camera = tde4.getCurrentCamera()
id_curve = tde4.getCameraZoomCurve(id_camera)

filename = tde4.postFileRequester("Select data file","*")
```

```
# open will throw if this fails
f = open(filename,"w");

id_key = tde4.getFirstCurveKey(id_curve)
while id_key != None:
        v = tde4.getCurveKeyPosition(id_curve,id_key)
        f.write("%f %f\n" % (v[0],v[1]));
        id_key = tde4.getNextCurveKey(id_curve,id_key)
f.close()
```

Script for exporting a zoom curve

It should be emphasized, that precision can be increased remarkably, if distinct lens objects are attached to primary and stereo camera, and if distinct zoom curves are calculated. The reason for this is the high degree of correlation mentioned before, which originates from the stereoscopic constraint.

3.2.6 *Parameter Adjustment*

It is natural that there are many unknown parameters involved in stereoscopic projects. In order to construct the worst case, let us assume you have two distinct lenses for either camera. Even in case of static focal length and static interocular we are left with the following scenario:

- primary focal length: 1 parameter
- primary lens distortion: 1 to 5 parameters
- secondary focal length: 1 parameter
- secondar lens distortion: 1 to 5 parameters
- interocular: 1 parameter
- vertical shift: 1 parameter
- depth shift: 1 parameter

which means, that in total, there can be 15(!) unknown parameters. It does not make sense to let 3DE4 calculate them simultaneously in the Parameter Adjustment Window. Moreover, it is better to group these parameters and calculate them separately. Our objective is to separate lens distortion calculations from stereoscopic. Let us recall the three methods for obtaining lens distortion (as demonstrated in our online tutorial)

1. Calibration shots (grids) are given and you use 3DE4's Distortion Grid Controls
2. Look for appropriate frames, which contain straight edges and use 3DE4's Distortion Grid Controls
3. Optimize lens distortion by means of 3DE4's Parameter Adjustment Window

For methods 1 and 2 lens distortion is not a stereoscopic issue, since 3DE4's Distortion Grid Controls work on single cameras. In a large stereoscopic production you hopefully have calibration shots for lens distortion, so we are left with a fine adjustment of two correlated focal lengths plus *interocular*, *vertical shift* and *depth shift*. In practice you can improve your result by grouping the parameters to be adjusted, e.g. like

primary focal, secondary focal, interocular
primary focal, secondary focal, vertical shift
primary focal, secondary focal, depth shift

interocular, vertical shift, depth shift

or you can try to do an adaptive adjustment on all parameters simultaneously.

For method 3, this document is still under construction. We assume that there is a certain degree of correlation similar to focal length, as we have seen in section 3.1.2. The recently implemented adaptive adjustment methods allow approaches we have not yet investigated in detail. It might be possible to do an adaptive parameter adjustment with all distortion parameters.

3.2.6.1 Brute Force vs. Adaptive Parameter Adjustment

If you are dealing with four or more parameters, you probably want to use the *adaptive* adjustment methods. In contrast to brute force, *adaptive* means that parameter space is not scanned as a grid. Instead, the optimizer uses the previous results in order to determine the subsequent iterative steps. However, adaptive methods can only be used, if there is a smooth relationship between the parameters and the pixel deviations. Some parameters are very appropriate for adaptive optimization, namely:

- interocular
- vertical shift
- depth shift

For these parameters adaptive optimization will usually work. Other parameters **can** be appropriate, but there are also counter examples which require a brute force adjustment, e.g

- primary + secondary lens distortion
- primary + secondary focal length

3.3 One Point Group (CamPG)

In dealing with stereoscopic projects we have to discuss scaling behaviour. Parameters like interocular, vertical shift and depth shift are of dimension "length", and this length is directly related to the point positions. Replacing the point model by another one which is scaled by, say, a factor 10 leads to interocular and shifts scaled by 10 as well. This means, if the stereo parameters represent real world lengths, so the distances between points do. Let us consider the relevant situations for the case of one point group:

- There are *survey data*. In this case the stereoscopic parameters are interpreted in the same space as the points of the point group. 3DE4 has no freedom to choose the scaling of the point group. There are no ambiguities to consider at all. All calculation modes for interocular and the shifts make sense. Alternatively, there may be a distance constraint between two points, which can be specified in

  Attribute Editor→Point Group→Distance Constraint

- There are *no survey data* (and there is no distance constraint). In solving the project, 3DE4 will choose the scaling of the point group in a way as to fit to the stereoscopic parameters. This is not very precise because the relative error the stereoscopic parameters directly translates to the scaling of the point group, but it's the only possibility we have. The lack of precision only affects scaling, i.e. the camera paths will be calculated correctly up to scaling. Therefore only the following modes for interocular make sense:

1. *static - user defined*: interocular is given by the user and 3DE4 will fix the point group scaling based on this information.
2. *dynamic - user defined*: this similar to *static - user defined*; for finding out the scale of the point group it doesn't matter if interocular is animated or not, as long as it's given by the user.
3. *static - calculated*: 3DE4 has no clue about any scaling, but will choose some point group scaling and calculate interocular in a way as to make it compatible to the point group. **This mode only works precisely if vertical shift and depth shift are zero.** Otherwise 3DE4 would be forced to calulate point group scaling as well as interocular from the shifts. We don't think this can be done seriously. If you are in this situation (no survey but non-zero vertical shift) consider ignoring vertical shift and set it to zero, or try to find a distance constraint for two points.

Without survey data nor distance constraint the mode *static - adjust* does not make sense for the following reason: Since scaling is not fixed, 3DE4 always has the freedom to compensate wrong interocular by scaling. In other words, we do not have a target function with a minimum. Therefore the dependency between deviation and interocular is a flat line.

3.4 One Point Group (ObjPG)

In some situations your single point group is not a camera point group but an object point group. In principle, for the core of 3DE4 the mathematics is very similar to the case of a camera point group. In non-stereoscopic projects the camera is located in the origin and looking towards $-z$. In stereoscopic projects however, we are dealing with a second camera as well, and this second camera is moving slightly in space as well. For 3DE4's database this means, there must be a way to store this camera movement. We solve this by adding an empty camera point group. So, in a proper setup you have the following point groups:

- *<some name 1>: Object, # points* (which contains all your tracking data)
- *<some name 2>: Camera, 0 points* (no tracking data, contains the camera movement)

After calculating, the primary camera is locked in the origin as expected and the secondary camera is located nearby and moving according to your specification (static or dynamic interocular, vertical shift, rotation policy). Everything else we said about projects with one camera in the previous section applies to this situation as well.

3.5 More than one Point Group (CamPG + ObjPG)

When there is more than one point group, i.e. a camera point group and one or more object point groups, scaling issues get more complicated. When there are survey data or distance constraints for all the point groups, there shouldn't be reason to worry. In all other situations, it might be helpful to have a certain understanding about how 3DE4 fixes the scaling of the point groups. Here are some rules which 3DE4 needs to apply for fixing the scale of the point groups.

- If a point group has survey data or a distance constraint, its scale is fixed.
- If *any* of the point groups has a fixed scale, 3DE4 uses this scale in order to fix the scale of all other point groups. For stereoscopic projects this is possible, because of the so-called *synchronicity relation* (see section below)
- If *any* of the point groups has a fixed scaling, 3DE4 will be able to calculate interocular and

vertical shift.

- If there is no point group with a fixed scale, then 3DE4 uses interocular to calculate the scaling of all point groups. This is possible, since there are two cameras with predefined geometric relation (we know their distance in space).
- If there is neither survey data nor distance constraint nor predefined interocular, 3DE4 fixes the scaling of the camera point group in some heuristic, hopefully reasonable way. All other scalings and interocular result from this ad-hoc scaling.

For the following considerations it does not matter, what calculation mode we choose for interocular or vertical shift (however, not *static - calculated*). Whatever modes are choosen 3DE4 will call the underlying core with some set of parameters, either once or by brute-force scanning or (in later releases) some adaptive mode. When calculation starts for such a given set of parameters, the core has to decide how to do the calculations, based on the following situations:

Survey data / distance constraints are available for all point groups

1. For all point groups the primary cameras are calculated by means of survey data
2. For all point groups the secondry cameras are calculated. All stereoscopic parameters are given.

Survey data / distance constraints are available for some of the point groups

1. For all point groups the primary cameras are calculated, either by means of survey data or using survey-free methods.
2. The core runs through all non-survey point groups. For each of these point groups a scaling factor is calculated and applied to the point group.

Survey data / distance constraints are not available for any of the point groups

1. For all point groups the primary cameras are calculated by means of survey-free methods.
2. For one of the point groups (preferably the camera point group) the scale factor is calculated. This includes calculating the secondary camera for this point group.
3. For all other point groups the scaling factor and the secondary camera are calculated.

3.5.1 *Trouble-shooting: Reducing the project*

When you are encountering problems with a stereoscopic project, there are several ways to reduce the number of objects to be calculated. The idea is to differentiate whether problems result from stereoscopy or from something else. You can:

- Reduce the number of point groups. Often for trouble-shooting it makes sense to run the stereoscopic with a single point group only. Stereoscopic projects with one point group provide fewer occasions to make mistakes. Point groups can be disabled in the *Object Browser*. This scenario is useful for getting an idea about camera parameters (focal length / zoom, lensdistortion) and stereoscopic parameters (interocular, vertical shift, secondary focal length / zoom).
- Reduce the number of cameras. For a stereoscopic project this means: disable the secondary camera (see *Object Browser*). This turns the entire project into a normal non-stereoscopic project. Now you can for instance let 3DE4 calculate the (primary) focal length / zoom, (primary) lens distortion etc. If this fails, it's not a stereosopic issue, because you have turned off stereo. If it works, calculate it and tweak it until your

experience with non-stereoscopic projects tells you know it's okay. Instead of disabling the secondary camera you can also disable the primary camera. If there are enough tracking data 3DE4 will treat this as a non-stereoscopic project.

If there are still problems, combine all methods and go back to a simple project with one camera point group. At some point it will become evident, what the problem is.

3.5.2 *The Synchronicity Relation (SyncRel)*

By SyncRel we understand a relation which is obvious in the real world, but has to be explicitly constructed by 3DE4. On the user side, this is simply a test if 3DE4 has calculated correctly. It occurs when there are **at least two point groups** and at least two synchronuous cameras, as in stereoscopic projects.
Let us consider an example. Assume you have a blue box and a moving object and you need to track both in stereoscopy. You get a camera point group and an object point group. When calculating all cameras, 3DE4 reconstructs the geometric relation between either of the two cameras and either of the point groups. For the camera point group this relation is simply the camera path. For the object point group, it's the motion of the camera viewed from the object. Combining these motions we get the motion of the object point group relative to the camera point group. The Synchronicity relation now says: this motion must be the same regardless if I construct it via primary or secondary camera. You can test this on 3DE4 by doing the following:

- Watch the object (point group) in 3DE4's 3D orientation panel
- Toggle between the primary and the secondary sequence

If you do so, the object point group should **not move**, not even a little. Otherwise, there might be a programming error or 3DE4 was for some reason overburdoned calculating the project. When this happens it might be helpful for user and developer to send the project to SDV's technical support. The Synchronicity relation should be fulfilled after each type of stereoscopic calculation, e.g. after postfiltering or finetuning.

3.6 Special Modes

Some projects are calculated by special methods within 3DE4's core. This mainly affects the so-called "nodal shots", i.e. shots with fixed camera position constraint when no survey data are given. The special thing about these projects is, that they do not lead to a reliable point model. In non stereoscopic projects, no real point positions are given, only directions with respect to the constant camera position. Point positions cannot be triangulated because the parallaxe is zero due to the constraint.

3.6.1 *Stereoscopy on Nodal Shots, no Survey*

In stereoscopic projects, parallaxe is not zero and we cannot neglect it, since this would not result in a true stereoscopic reconstruction of the camera rig. On the other hand, the parallaxe is small, since it is dependent on the ratio of interocular and convergence; the former is often very small in comparison to the latter. So, a point model based on the triangulation of a stereo pair will generally be less precise and reliable than a point model triangulated from an constrainted single camera. For this reason we decided to implement this as special mode in 3DE4. This mode is activated when your project fulfills the following conditions:

- There are exactly two cameras and they form a stereoscopic pair.
- Both have the *Fixed Camera Position Constraint* turned on
- There is exactly one point group and it is a camera point group

The rotation policies *y* and *xyz* can both be used in this mode. Please do not provide survey data for the points.

3.6.1.1 Parameter Adjustment

Since the scaling of the point group is not fixed, it does not make sense to do a parameter adjustment with interocular. If you do not know it, we recommend to set it to some reasonable value, like the default 6.5cm. With a fixed interocular it might be possible to adjust vertical shift and depth shift, although we have notice from user projects, that there is not enough parallaxe information to do so.

3.6.1.2 Current state of development

In the present implementation the common pivot is exactly in the middle between primary and secondary camera with respect to interocular, vertical shift and depth shift. 3DE4 constructs a solution by placing this pivot point in the origin. Calculating a zoom curve is not possible.

3.7 Stereoscopy in relation to other functions of 3DE4

3.7.1 *Interpolation*

Frames which have no or insufficient tracking data are generally handled by 3DE4 by means of interpolation. For non-stereoscopic projects this affects frames with three or less tracked points. 3DE4 uses all available information to locate the camera in relation to the point group. In stereoscopic projects, we have to define, in how far under-determined frames are handled. The current state of development is the following: the minimum requirements for locating the camera in a given frame is four tracked points for the primary camera and two tracked points for the secondary camera. The following tables explain, in how far camera positionand rotation are well-defined in a frame for a given number of tracked points in that frame. We get a different behaviour for the primary and the secondary camera. In detail, for the primary camera we have:

Number of tracked points	Position	Rotation
4 or more	unique	unique
3	unique*	unique
2	interpolated	unique
1	interpolated	1 degree of freedom
0	interpolated	interpolated

For three points the position is interpolated first, but will be fixed uniquely in a second step. The secondary camera is bound by the stereoscopic constraint. Therefore we need less tracking information in order to calculate it. For *rotation policy xyz* we have

Number of tracked points	Position	Rotation
2 or more	unique	unique
1	not precise	1 degree of freedom

Number of tracked points	Position	Rotation
0	**not calculated!**	**not calculated!**

For *rotation policy y* we have:

Number of tracked points	Position	Rotation
1 or more	unique	unique
0	**not calculated!**	**not calculated!**

For the secondary camera it is quite important to have at least one tracked point, since the current implementation does not interpolate when there are no tracking data. Generally, the values we present here are *theoretical boundaries*. It should be emphasized that usually more tracking data are required to get a robust calculation and precise results.

3.8 Current State of Development

3.8.1 *Known Bugs*

In some situations it may happen that not all points of some point group are calculated. This happens when 3DE4 decides there is not enough parallaxe. The internal condition of 3DE4 whether or not points are calculated are quite paranoid. It is unclear at the current state of development at what stage of processing these points can be calculated without endangering other parts of the project.

In stereoscopic projects with more than one surveyed point group, the interocular calculation mode *static - calculated* may lack precision, since only one point group is used to calculate interocular. Please use *static - adjust* instead.

3.8.2 *Unknown Bugs*

Every stereoscopic project (working or not) is welcome. Please send them to uwe@sci-d-vis.com. Footage is usually not required. 3DE4-projects do no contain confidential data from your project. Thank you!

A *Mathematics*

In this section we nail down the definitions of the stereoscopic parameters. If you are not familiar with mathematics, please read this section nevertheless and just skip the formulas but read the **notes typeset in bold**. We use the canonical unit vectors

$$e_x = (1,0,0)^T$$
$$e_y = (0,1,0)^T$$
$$e_z = (0,0,1)^T$$

For any two vectors a and b, $\otimes$ denotes the tensorproduct

$$(a \otimes b)_{ij} = a_i \, b_j$$

of these two vectors.

A.1 Interocular, vertical shift, depth shift

Without loss of generality we assume that the primary camera is left-hand and the secondary is right-hand. Now, let m_L be the orientation matrix of the left (i.e. primary) camera and p_L its position. Then

$$c_{L,y} = m_L\, e_y$$

is the up-vector of this camera and

$$c_{L,z} = m_L\, e_z$$

its optical axis. Note, that we choose a definition without sign for the optical axes, which means the axes are pointing backwards, but our definition of depth shift will become more intuitive. We define the *stereoscopic plane* as the unique plane perpendicular to $c_{L,y}$ and containing the primary camera position p_L. Now we consider the secondary camera at position p_R with orientation m_R, optical axis $c_{R,z} = m_R\, e_z$. Due to our general rotation policy, $c_{R,z}$ does not necessarily lie within the stereoscopic plane. Therefore, our first step is to project it into this plane. Let

$$\Pi^{\perp}_{\text{stereo}} = \mathbf{1} - c_{L,y} \otimes c_{L,y}$$

be the projector, which projects vectors into the stereoscopic plane. Then we define the *projected secondary optical axis* by

$$c'_{R,z} = \text{unit}(\Pi^{\perp}_{\text{stereo}}\, c_{R,z})$$

Also, we project the secondary camera position into the stereoscopic plane:

$$p'_R = \Pi^{\perp}_{\text{stereo}}\,(p_R - p_L) + p_L$$

Now we get a straight forward definition of *vertical shift* (which we denote d_{vert} in the following):

$$\boxed{d_{\text{vert}} = c_{L,y} \cdot (p_R - p'_R)}$$

Vertical shift is positive, when the secondary camera is located higher than the primary camera. For *depth shift* we consider the unit vector c_{center} which we call the *central axis of view* that bisects the two (projected) optical axes $c_{L,z}$ and $c'_{R,z}$:

$$c_{\text{center}} = \text{unit}(c_{L,z} + c'_{R,z})$$

Clearly, c_{center} lies in the stereoscopic plane. We define *depth shift* (called d_{depth} in the following) by

$$\boxed{d_{\text{depth}} = c_{\text{center}} \cdot (p'_R - p_L)}$$

Roughly speaking, depth shift is positive, when the secondary camera is located *behind* the primary camera. This makes sense, since the positive z-direction in camera space points backwards. Note, that the d_{vert} and d_{depth} interpreted as vectors instead of distances are perpendicular to each other. Let

$$\Pi^{\perp}_{\text{center}} = \mathbf{1} - c_{\text{center}} \otimes c_{\text{center}}$$

be the projector into the plane perpendicular to c_{center}. Then it's clear how to define *interocular*:

as a vector it is perpendicular to both d_{vert} and d_{depth}. As a (signed) distance it reads:

$$d_{\text{ioc}} = c_{\text{center}} \cdot \Pi^{\perp}_{\text{center}} (p'_{\text{R}} - p_{\text{L}})$$

In other words, when the secondary camera is located *right* to the primary camera, interocular is positive. However, 3DE4 allows (or demands) explicit specification on which side the primary camera is located. We have already discussed this in section 3.2.1.2. What we have been doing here is to constuct a coordinate system from the primary camera up-vector $c_{\text{L},y}$, and the central axis of view c_{center}. This coordinate system is related to the corrdinate system of the primary camera by a rotation around e_y by half the parallaxe. Our parameters *interocular* and the two shifts correspond to the difference vector of the camera positions expressed in this coordinate system.

A.2 Parallaxe and Convergence Point

By this occasion we can also define some more quantities. In the current release of 3DE4 the convergence point is a *backend quantity*: it can be calculated, but not be pre-defined, and it has no impact on the calculation. Unless requested by the user, this will be the case in future releases as well. The *parallaxe* can be interpreted as angle between the projected optical axes:

$$\alpha = \text{atan2}(\| c_{\text{L},z} \times c'_{\text{R},z} \|, c_{\text{L},z} \cdot c'_{\text{R},z})$$

The following is the definition of the *convergence point* we use in 3DE4, regardless of presence or absence of *vertical shift* (and *depth shift*) and for all rotation policies. This is not the only way to define the convergence point. Another possibility is to use the projected optical axes instead of the real (possibly skew) optical axes. Our definition is symmetric with respect to primary and secondary camera. Any ray in three-dimensional space is given by a pair (p,u) where p is the start point of the ray and the unit vector u its direction. Given two camera positions p_{L} and p_{R} and the non-parallel corresponding optical axis' u_{L} and u_{R} we define the *convergence point* as the point p_{conv} which minimizes the sum of squares of distances to $(p_{\text{L}},u_{\text{L}})$ and $(p_{\text{R}},u_{\text{R}})$. This is calculated as follows. For any unit vector u in $\mathbf{R}^3$, $||u|| = 1$ let

$$\Pi^{\perp}_{u} = \mathbf{1} - u \otimes u, \; ||u|| = 1$$

be the projector $\mathbf{R}^3 \rightarrow \mathbf{R}^3$ which projects any vector into the plane perpendicular to u. Then our definition of the *convergence point* reads

$$p_{\text{conv}} = (\Pi^{\perp}_{u_{\text{L}}} + \Pi^{\perp}_{u_{\text{R}}})^{-1} (\Pi^{\perp}_{u_{\text{L}}} p_{\text{L}} + \Pi^{\perp}_{u_{\text{R}}} p_{\text{R}})$$

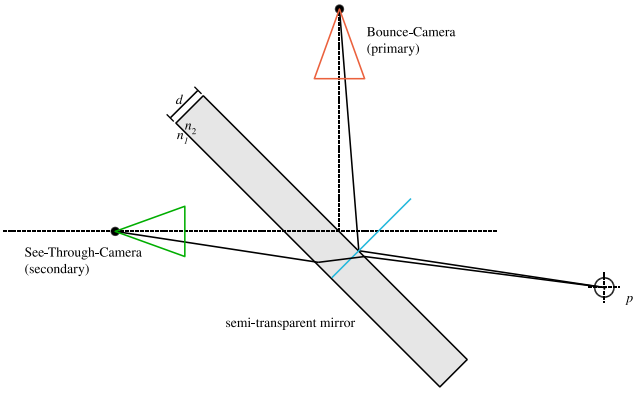
We omit the proof that this point in fact minimizes the distance to the optical axes as mentioned above (takes a few minutes to do).

B *Semi-transparent mirror*

At least one brand of camera rigs used in practice allows a particular geometry for the primary and secondary camera, in which the primary camera records the scene via reflection at a semi-transparent mirror, while the secondary camera records the scene by transmission at this mirror. The question arises, in how far refractive effects of this mirror affect precision in stereoscopic matchmoving. The analysis and examples in the following sections show, that in fact precision is affected when dealing with semi-transparent mirrors. The next question is, how this should be handled within 3DE4. We do not have notice that there are problems with the bounced

camera, so everything we do in the following is about the secondary camera. For our simulations we used a refractive index of 1.5. This is not the worst case since there are glasses with an even higher index. Yet, we hope the mirror used in practice has a lower index, since this reduces the undesired effect.

The discussion was initiated by Lafleche Dumais from *Hybride* - Digital Visual Effects, Canada.

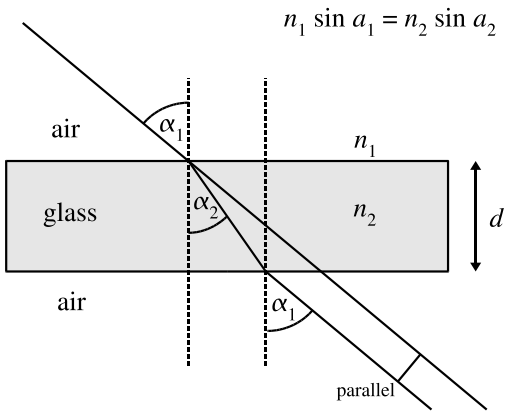


B.1 Refraction

It is not important here that we are dealing with a semi-transparent mirror. The point is, that the secondary camera is looking through a planar glass plate (with a very thin reflective layer on it). This glass plate *displaces* any ray of light passing through. It's direction outside the glass plate remains unchanged. We recall the Snellius Law of Refraction:

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{n_2}{n_1} \tag{1}$$

where n_1 and n_2 are the refractive indices of the two adjacent materials (air and glass). α_1 and α_2 are the angles between the ray and the plane normal in medium 1 (air) and medium 2 (glass), respectively. Using the refraction law we will model the effect of the glass plate on the recorded image.



Snellius refraction law

B.2 Modelling

We will do the calculations in the coordinate system of the camera, i.e. the camera is located in the origin and looking towards negative *z*-direction. Given an object point *p* recorded by the camera with no glass plate, its direction with respect to the camera position is the unit vector from the camera position to *p*:

$$u = \mathrm{unit}(p) \tag{2}$$

The aim is to calculate a modified direction *u'* which represents the point direction when the camera looks through the glass plate. Let *n* be the normal vector of the refractor and

$$q = \frac{n_1}{n_2} \tag{3}$$

the ratio of the refractive indices. We split the position of *p* into a *normal* part (parallel to *n*) and a tangential part (perpendicular to *n*):

$$p = n(p \cdot n) + t(p \cdot t) \tag{4}$$

where the normalized tangential vector *t* is

$$t = \mathrm{unit}\,(p - n(p \cdot n)) \tag{5}$$

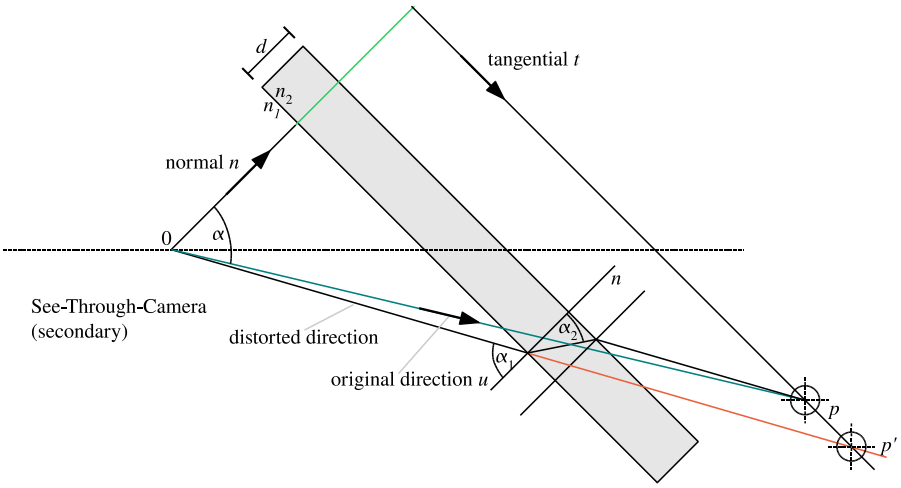
It will be useful later to express *p* by the angle with respect to *n*.

$$p = (p \cdot n)(n + t \tan\alpha) \tag{6}$$

where

$$\tan\alpha = \frac{p \cdot t}{p \cdot n} \tag{7}$$

So far we have defined the situation without the glass plate. If we introduce the refractor, the following question arises: How does the light travel from point *p* to the camera?



Outside the glass, the ray has an angle of α_1 with respect to *n*, inside the angle is α_2. We can now describe the tangential part by means of these angles by summing up the ways outside and inside the glass:

$$p \cdot t = (p \cdot n - d)\tan\alpha_1 + d\tan\alpha_2 \tag{8}$$

The two angles are related by Snellius' law. If we use $\cos^2\alpha_2 = 1 - \sin^2\alpha_2$ and replace α_2 we get:

$$p \cdot t = (p \cdot n - d) \tan \alpha_1 + d \frac{q \sin \alpha_1}{\sqrt{1 - q^2 \sin^2 \alpha_1}} \tag{9}$$

α_1 represents the direction of p when the camera looks through the glass plate. Let us exmine this result and check, if it shows a reasonable behaviour. We can expect, that this equation becomes trivial when either $d = 0$ (there is no plate) or $q = 1$ (there is no refraction). In fact, in either case we get

$$p \cdot t = p \cdot n \tan \alpha_1 \tag{10}$$

which is correct. The above equation for α_1 not very easy to solve analytically, since it leads to a quartic equation in $\sin \alpha_1$ (and we didn't try). The easiest way is to reformulate it as a fixed-point problem and solve it numerically by two or three iterations:

$$\alpha_1^{(k+1)} = \arctan \left[\frac{1}{p \cdot n - d} \left(p \cdot t - d \frac{q \sin \alpha_1^{(k)}}{\sqrt{1 - q^2 \sin^2 \alpha_1^{(k)}}} \right) \right] \tag{11}$$

where we get a good initial value from

$$\alpha_1^{(0)} = \alpha \tag{12}$$

This initial value reflects the fact that the effect of the refractor is quite small. Now we can construct some kind of *vertex shader* function f by replacing p by a new point p'. We do this by shifting p along the tangential vector t.

$$p' = f(p) := (p \cdot n)(n + t \tan \alpha_1) \tag{13}$$

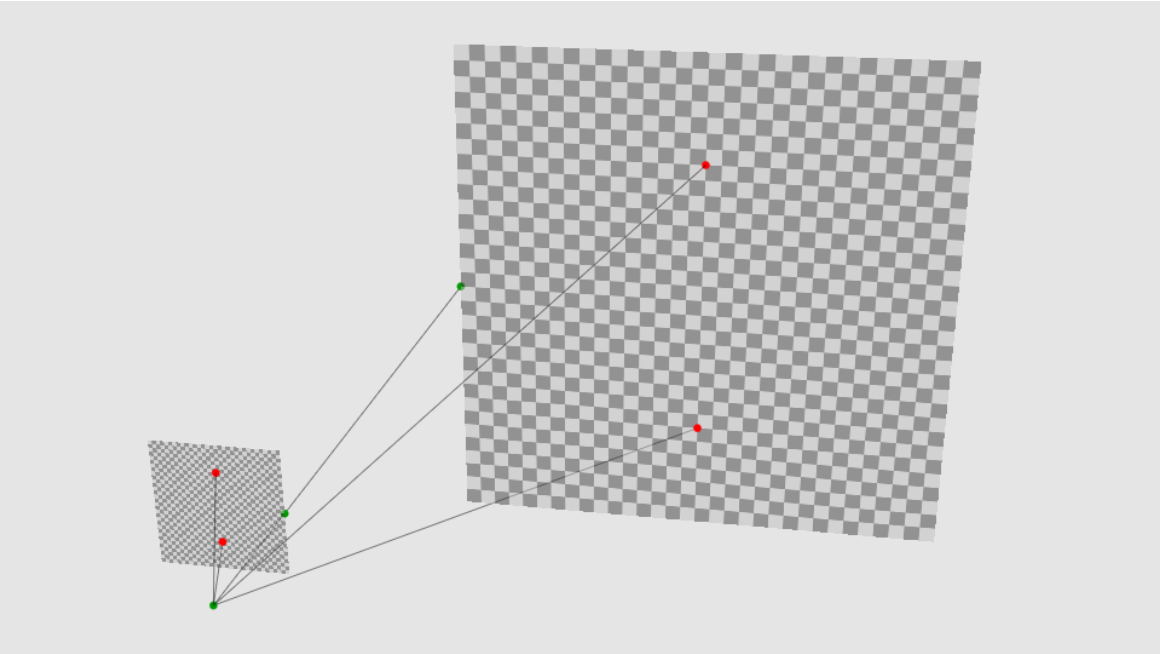
which completes our calculations. The shader function has a fixed point, namely at $(p \cdot n)n$, which means $\alpha = \alpha_1 = 0$:

$$p' = p = f(p) := (p \cdot n)n \tag{14}$$

This fixed point is visualized in section B.4 where we investigate the (theoretical) limits of the mirror construction. Note, that we had some freedom in choosing the position p'; we could have used a different distance to the camera as well. Our shader function f does not reproduce normal vectors correctly (because we did not explicit construct it to do so). A detailed analysis would probably show that the ambiguity in choosing the distance of p' can be utilized to implement the correct behavour of the shader function with respect to normal vectors. However, f as constructed above provides all information we need in the following.

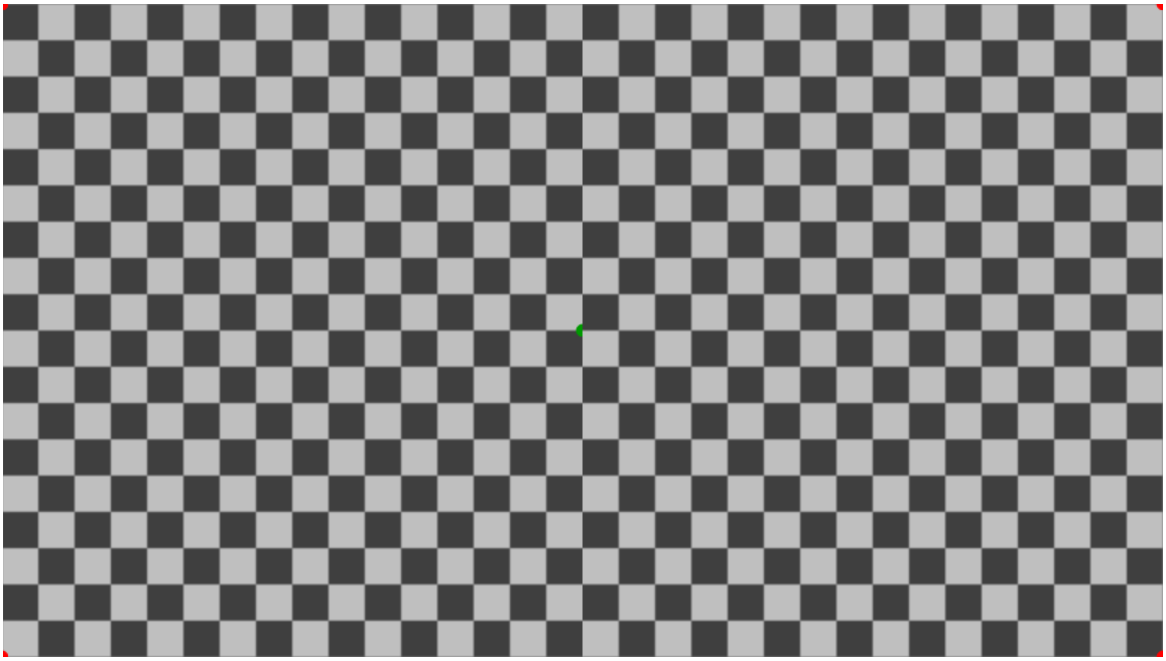
B.3 Examples

For our first example we use the following setup. We have placed a camera with filmback 1.6cm / 0.9cm and focal length 2.0cm in the coordinate origin. The two checker grids are located at 100cm and 500cm distance, perpendicular to the z-axis. The grids are arranged in a way that the closer one covers the left half of the image to be rendered, while the afar one covers the right half. We choose this setup in order to demonstrate the relation between the distance of the recorded object and the strength of the distortion effect caused by the glass plate.



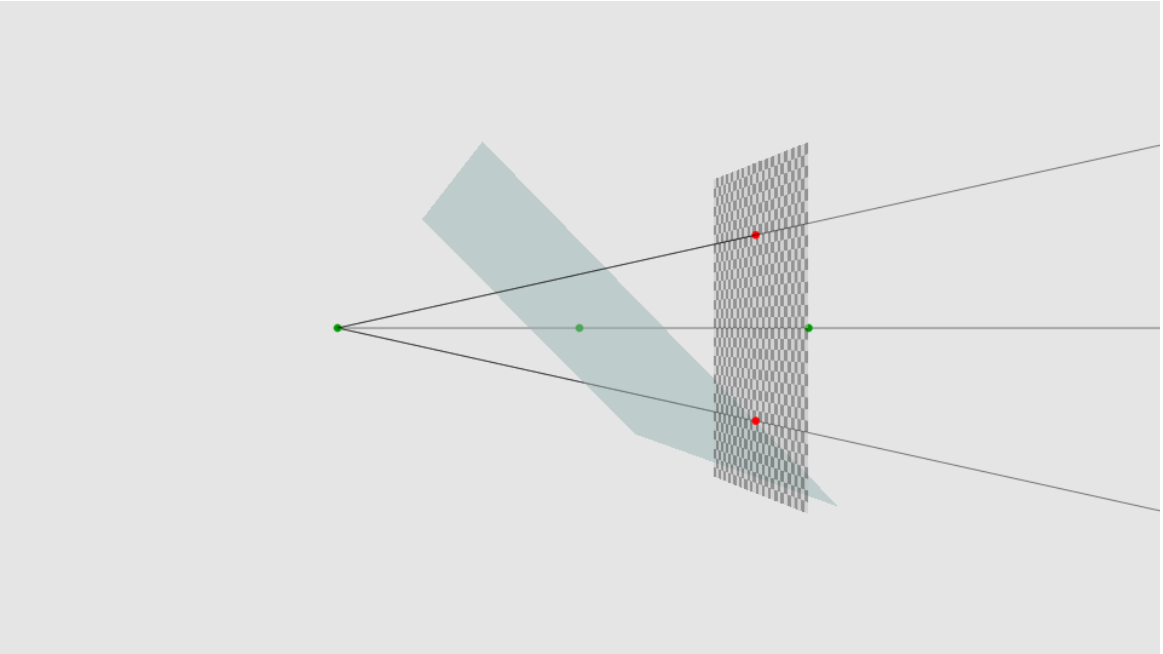
Two grids at 100cm and 500cm distance to camera

The resulting image is:

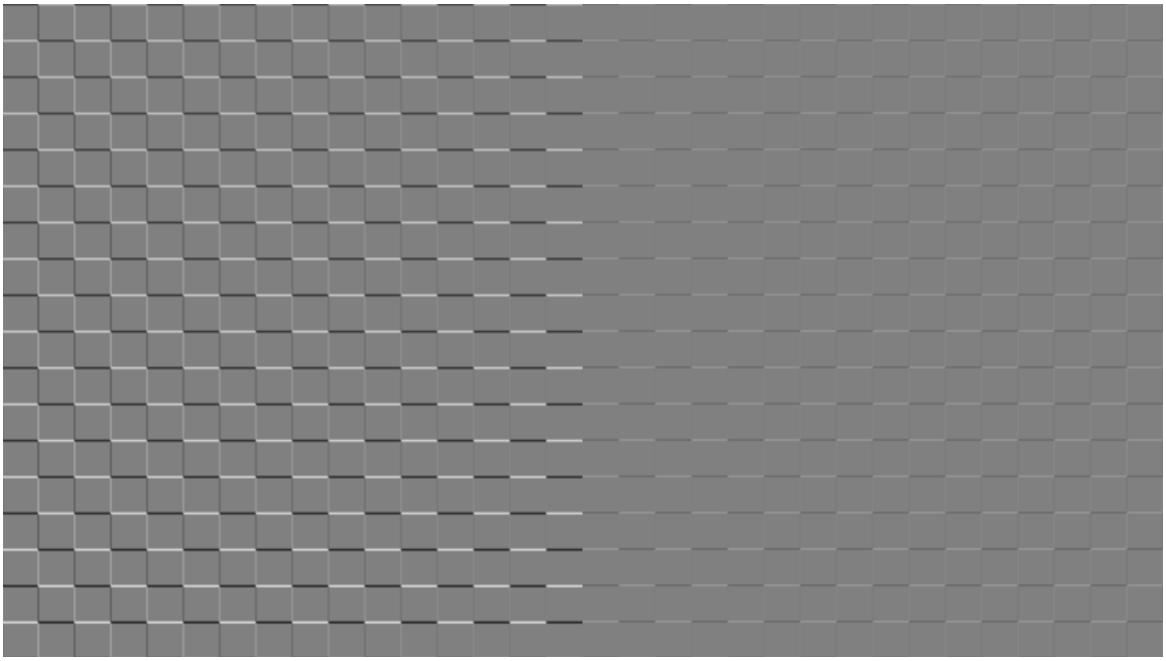


Undistorted image of two grids at 100cm and 500cm distance to camera

Now we simulate a glass plate, e.g. a semitransparent mirror. The refractive index is 1.5, and its thickness is 0.25cm. Its normal vector points towards (0,1,-1).



We calculate the difference between the undistorted image and the image distorted by the glass plate. The result is:



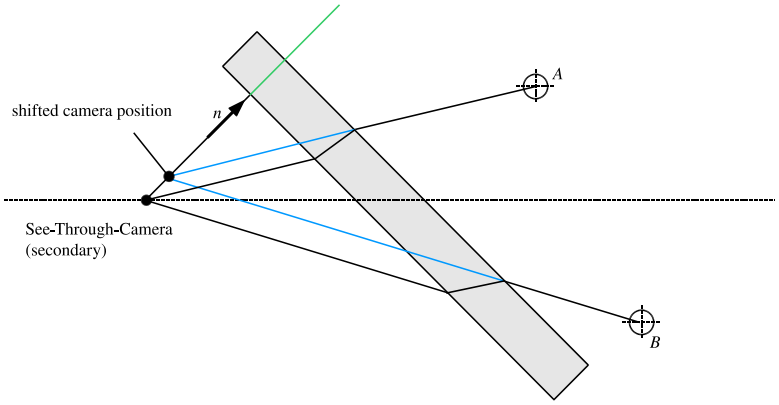
Difference image: undistorted vs. glass plate distortion

Note, that this image has only a resolution of 960x540 pixel. For a typical 16:9-plate of 1920x1080 the effect is twice as high. From this image, we obtain the following:

1. There is a noticeable effect from the glass plate, and its order of magnitude is 1 to 2 pixel.
2. As expected, the effect depends on the distance of the recorded object. As the distance becomes larger, the effect disappears.

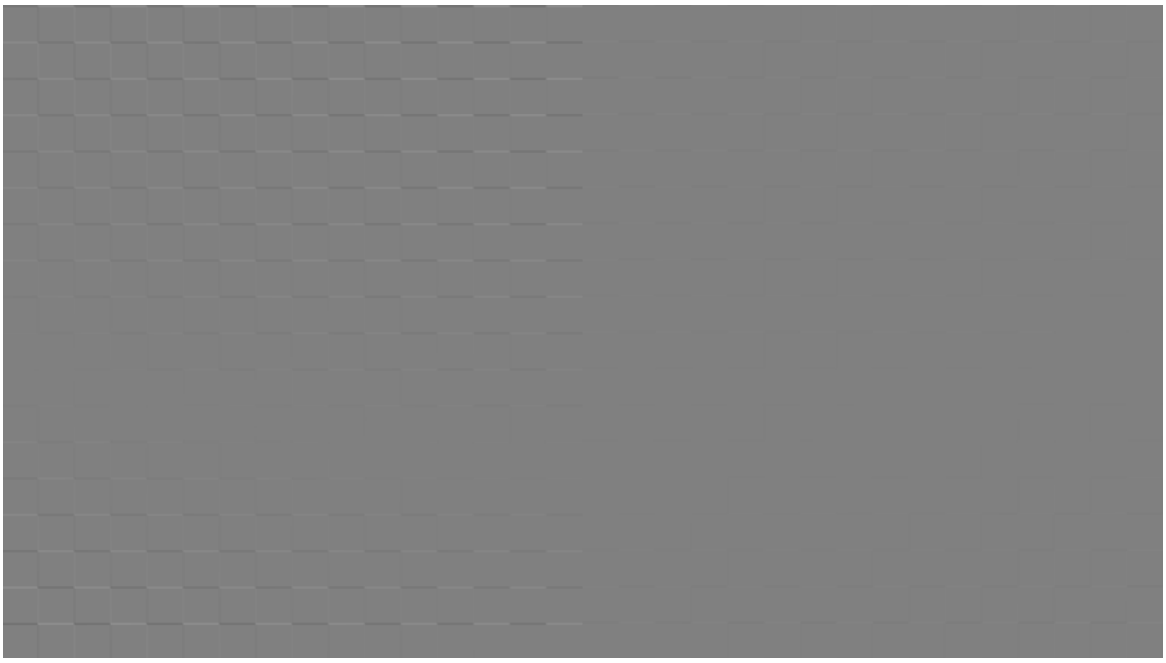
What we see in the image can be considered as a mismatch between primary and secondary camera. The question is now, how to deal with it. Since the effect depends on the distance of the

recorded object, there is no two-dimensional undistort-operator which could remove the effect; whenever you handle the closer grid you get in trouble with the afar one and vice versa. For the development of 3DE4 this means: we cannot modify our lens distortion model in order to handle the effect since lens distortion is a purely two-dimensional image processing method. The same applies to modelling the effect by a zoom around the fixed point. On the other hand,we are reluctant to implement the shader function we have discussed before since this would have a huge impact on the structure of our calculation core. Therefore, our proposal is to compensate the glass plate by an effective stereo rig model which involves a combination of depth shift and vertical shift, as shown inthe figure below.



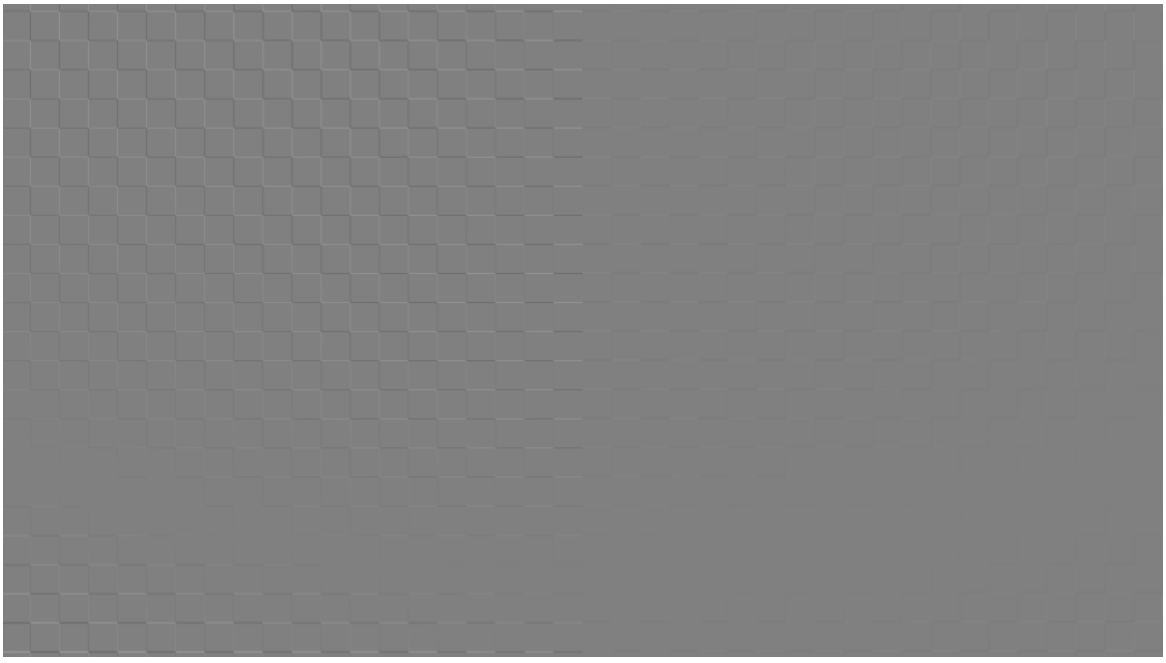
Compensating the mirror by shift

By doing so, we do not get rid completely of the effect, but we can compensate it to a certain degree, as demonstrated in the following. Also, this method seems to work for different distances, like 100cm and 500cm. In practice, finding depth shift and vertical shift is done by 3DE4's parameter adjustment, as described by several occasions in this document. Also, according to our experience and discussions with users depth shift and vertical shift have values much higher than those imposed by the glass plate, so that often it will not be possible to tell apart which fraction of vertical and depth shift results from which mechanical property of the rig. But for the following, we concentrate on the glass plate.



Difference image: shifted camera position vs. glass plate distortion

For this image we have rendered the difference between the glass plate image as described before and an undistorted image with a camera position shifted by (0,0.0883,-0.0883). The shift vector was determined by try-and-error. For the given parameters, the difference clearly is less than 1 pixel for the 100cm-grid. We did the same procedure for another setup. For the following image we used a camera with filmback 4.0cm/2.25cm and a focal length of 2.0cm, i.e. the horizontal aperture angle was 90 degree. The camera was shifted by (0,0.12,-0.12). The difference between the glass plate image and the shift position image is:

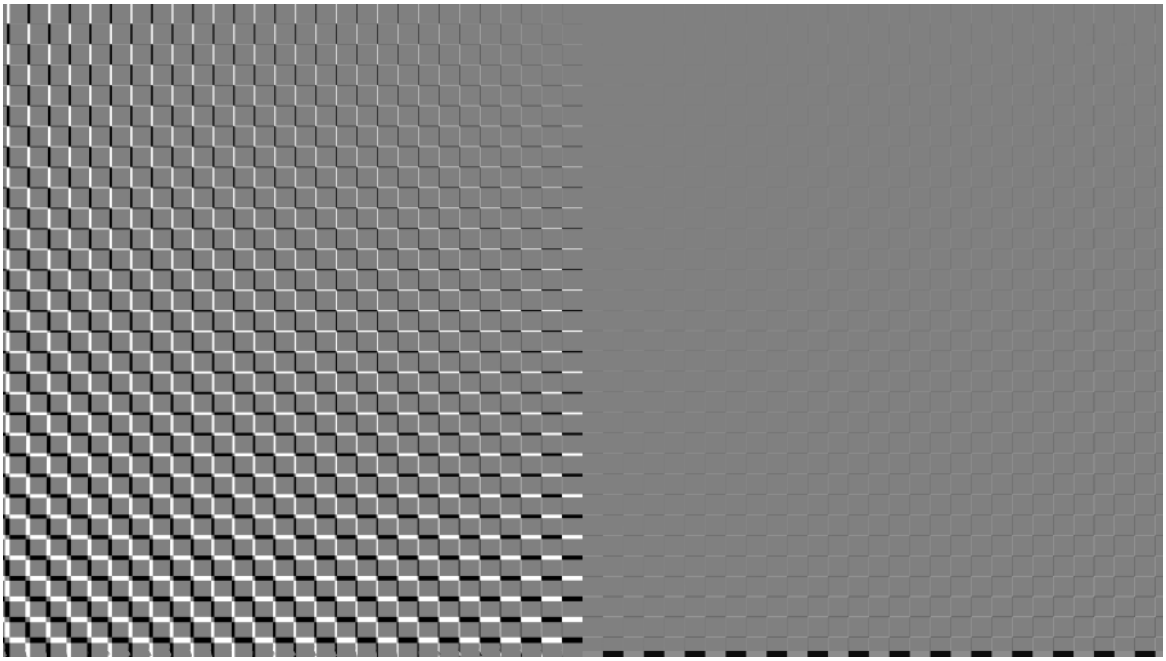


Difference image: shifted camera position vs. glass plate distortion at 90 degree horizontal angle

So, our conclusion is that at least according to our examples depth shift and vertical shift are appropriate for compensating the glass plate.

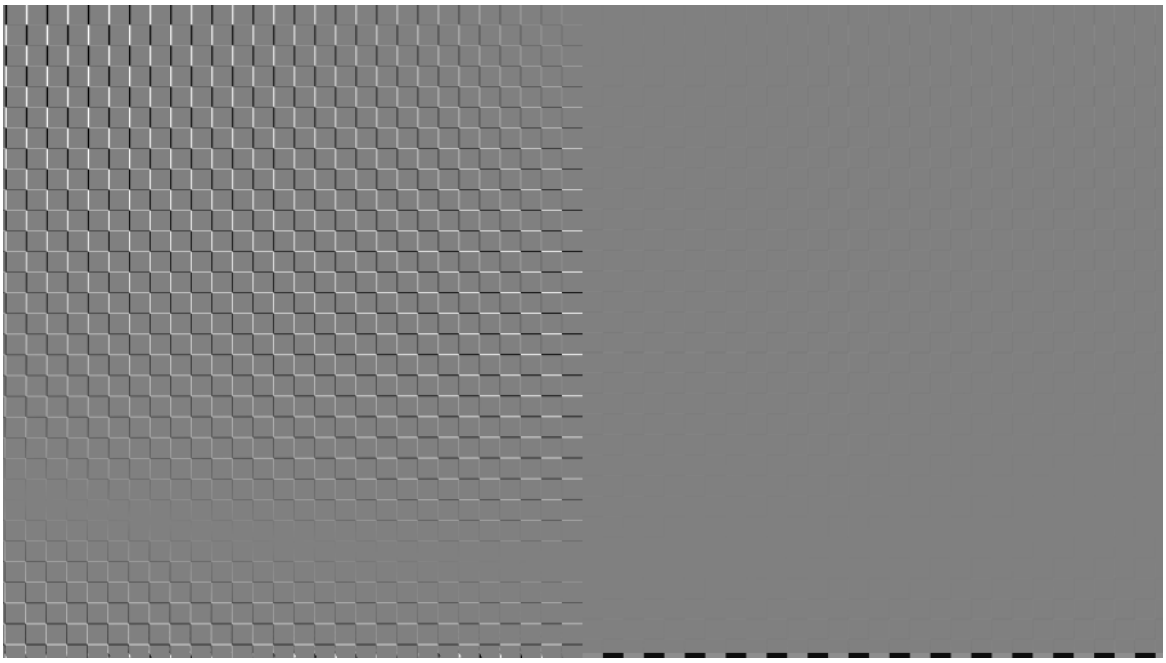
B.4 Limits

In practice the semi-transparent mirror has an angle of 45 degree with respect to the optical axis. This restricts the vertical aperture angle to 90 degree. However, the transmissivity of the mirror will decrease dramatically at the edge of the image, i.e. when α approaches 90 degree. The exact relationships is given by the →Fresnel equations and we will not discuss it here. Nonetheless, it might be interesting to see the distortion effect when the vertical angle tends to 90 degree. In the following example we have done this. Also, the near grid has now a distance of 20cm instead of 100cm. At the bottom of the image you see artefacts from the collapse of our iterative shader function. In the middle of the top border you see the fixed point of the shader equation, where the effect vanishes.



Difference image: undistorted vs. glass plate distortion at a vertical angle of 89.85 degree

Here, again, the compensation by shifting the camera position by (0,0.15,-0.15). Clearly, the shifting technique is challenged for this extreme case. Nevertheless, we can observe an improvement from 5.0 pixel to 1.0 pixel at 1k resolution.



Difference image: shifted camera position vs. glass plate distortion at a vertical angle of 89.85 degree

B.5 Primary camera looking up

In case the mirror is placed as to reflect incoming light downwards like in the figure below, every rendered image presented in the previous sections has to be flipped vertically. Instead of shifting the camera like (0,*d*,-*d*) the shift then reads (0,-*d*,-*d*).